

# Optimized structure of two layered porous media with genetic algorithm for transpiration cooling

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## Abstract

The genetic algorithm (GA) based on an analytical solution of a simplified local thermal non-equilibrium model is presented to optimize a porous structure which consists of two layered media and is used as a carrier of transpiration cooling in this paper. Fluid coolant is injected with a certain pressure difference and coolant mass flow rate respectively into the pores of the structure. The lowest temperature at the hot surface to be protected from a high heat flux is seen to be the target of optimization under the global constraints of weight and cost. The optimal composition, porosity and thickness of the two layers emerge from an entire iteration process. The results indicate that the thermal conductivity and porosity of the second layer near the hot side is very important for the hot side temperature. Simultaneously the thermal conductivity of the first layer near the cold side has neglected influence on the hot side temperature, and only under the condition of a certain pressure difference, the porosity of which can distinctly influence on the hot side temperature.

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*Keywords:* Layered porous structure; Transpiration cooling; Genetic algorithm; Optimization

## 1. Introduction

With the development of spaceflight and aviation technology, transpiration cooling is widely used in various regions, such as hypersonic vehicle combustors, gas turbine blades, cryogenic liquid rocket engines, and others. The ultimate goal of the transpiration cooling system design is to obtain an allowable temperature at the hot surface of the structure loading a high heat flux. In most investigations on transpiration cooling, the porous structure was seen to be homogenous or made of a single layer. Greuel et al. [1], Landis and Bowman [2] studied cooling performance of the homogenous structures, and indicated that the thermal conductivity, porosity, characteristic size and thickness of the porous structures can influence on the hot surface temperature. It is therefore difficult to describe the relationship between the hot surface temperature and these factors with a simple function.

Wolfersdorf [4] investigated cooling performance of a certain porous structure with two layered media, and presented an analytical solution to calculate the cooling effectiveness. Lee et al. [3] registered a patent to combine various porous and non-porous layers in a compliant cooling structure. Multi layered porous structures have some distinct advances to optimize cooling and decrease cost, but it is really a new challenge for the investigators of transpiration cooling system, how to design the multi layered structures under the mutual constrains of the composition, thickness and porosity of each layer, total weight and cost.

As a highly parallel, random and self-adapting searching algorithm developed in the 20th century, the genetic algorithm (GA) has been successfully applied in many intelligent optimization problems. In the region of heat transfer optimization, there are also some successful instances. Based on the local thermal equilibrium model, Wildi-Tremblay and Gosselin [5] studied on structure optimization of a stack of porous layers through which a coolant flows, the optimized factors included the composition, porosity of each layer, total weight and cost of the entire porous structure. Ozkol [6] showed shape optimizations of mono- and two-dimensional structure obtained by

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**Nomenclature**

$y, Y$	coordinate
$H$	thickness of entire structure . . . . . m
$h$	thickness of the first layer . . . . . m
$T$	temperature . . . . . K
$k$	thermal conductivity . . . . . W/m K
$h_v$	convective coefficient . . . . . W/m <sup>3</sup> K
$Be$	Bejan number
$K$	permeability . . . . . m <sup>2</sup>
$m$	Coolant mass flow rate . . . . . kg/m <sup>2</sup> s
$d_p$	characteristic size of porous matrix . . . . . m
$c$	specific heat capacity . . . . . J/kg K
$\mu$	viscosity . . . . . N/s m <sup>2</sup>
$\alpha$	thermal diffusion coefficient . . . . . m <sup>2</sup> /s
$Bi_v$	Biot number within pores
$Bi_c$	Biot number at cold surface

$M$	dimensionless coolant flow rate
$Re$	Reynolds number

*Greek symbols*

$\varepsilon$	porosity
$\kappa, \lambda$	effective conductivity ratio
$\theta$	dimensionless temperature

*Subscripts*

$s$	solid
$f$	fluid
$c$	coolant reservoir
$g$	hot gas
$i$	layer number
$e$	effective

a new GA which was improved to exhibit a good convergence toward the optimal solution. Younes [7] presented a design of optimal heat exchanger in an energy-converting system, and the geometry, number of heat transfer units and pressure drop were determined by the GA. However, it is necessary to study the optimization problems of the multilayer structure for transpiration cooling.

In this paper, an analytical solution of simplified local thermal non-equilibrium model is used to solve the fitness of the GA chromosome of the structural optimization, and optimizations of two porous layers are carried out at a certain pressure difference and coolant mass flow rate respectively. The optimized factors include the composition, porosity and thickness of each layer, and the results obtained without or with weight and cost constrain are discussed. The objective of this work is to provide to the investigators of cooling technique with an intelligent method to design transpiration cooling structures.

**2. Mathematic model**

A physical sketch of transpiration cooling is shown in Fig. 1. Fluid coolant is injected into a porous structure with a mass flow rate of  $m$  at a reservoir temperature of  $T_c$  to protect the hot surface of the structure from a high heat flux of  $q$  thereby. The porous structure consists of two layered media, and the composition, porosity and thickness of each layer may be different. The internal structure of porous media is assumed as packed bed. Generally, the Reynolds number ( $Re = \rho_f u d_p / \mu_f$ ) of pores in transpiration cooling is in a magnitude of one or two orders [1], the inertia forces are therefore not negligible anymore. For this steady, incompressible and one-dimensional problem, the coolant flow satisfies the following momentum equation:

$$\nabla p = -\frac{\mu_f}{K_i} u - \frac{\rho_f F_i}{\sqrt{K_i}} |u|u, \quad i = 1, 2 \tag{1}$$

Here,  $K_i$  and  $F_i$  are the permeability and Forchheimer coefficient of each layer, and calculated by the formulae suggested

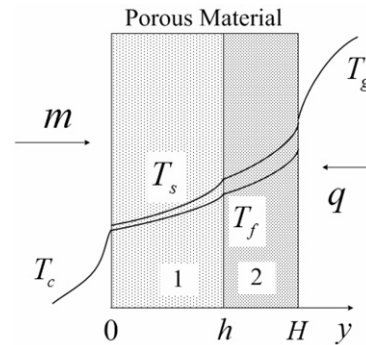


Fig. 1. Schematic diagram of transpiration cooling problem of two porous layers.

by Alazmi and Vafai [8]:

$$\begin{cases} K_i = \frac{d_p^2 \varepsilon_i^3}{150(1-\varepsilon_i)^2} \\ F_i = \frac{1.75}{\sqrt{150\varepsilon_i^3}} \end{cases} \tag{2}$$

When the pressure difference between the cold and hot sides is assumed to be constant, any variations in the porosity and thickness of each layer could lead to a change of coolant mass flow rate. Bejan [9] suggested the following dimensionless number to estimate the power of driving the fluid coolant through the porous structures:

$$Be = \frac{H^2 \Delta P}{\alpha_f \mu_f} \tag{3}$$

Then,  $Be$  is called Bejan number.

When the coolant mass flow rate is assumed to be constant, it is not necessary to consider the momentum equation, and dimensionless coolant mass flow rate  $M$  can be directly obtained by:

$$M = \frac{m c_f H}{k_f} \tag{4}$$

When the thermal conductivity of the porous structure is much larger than that of coolant, but the porosity is not very

large, the effective conductivity ratio of solid structure to fluid coolant is much larger than 1, Wolfersdorf [4] as well as Wang and Wang [9] neglected the thermal diffusion of the fluid coolant, and the local thermal non-equilibrium model was simplified as:

$$k_{se,i} \frac{d^2 T_s}{dy^2} = h_{v,i}(T_s - T_f) \quad (5)$$

$$m c_f \frac{dT_f}{dy} = h_{v,i}(T_s - T_f) \quad (6)$$

Here,  $k_{se,i} = k_{s,i}(1 - \varepsilon_i)$  is the effective thermal conductivity of the solid material of each porous layer,  $h_{v,i}$  is the volumetric heat transfer coefficient of the convection within the pores of the porous structure. It is clear that the diversity of the heat transfer coefficient will lead to different optimal results. Therefore, it is necessary to know the intrinsic architecture of porous media before optimization. When the Reynolds number is larger than 350, Hwang [10] suggested the following formula to calculate the heat transfer coefficient:

$$h_{v,i} = \frac{20.346(1 - \varepsilon_i)\varepsilon_i^2}{d_p} \left[ 1.064 \left( \frac{k_f}{d_p} \right) Pr^{0.33} Re^{0.59} \right] \quad (7)$$

To solve the simplified local thermal non-equilibrium model, Eqs. (5)–(6), Wang and Wang [9] used the following boundary conditions at the cold and hot sides, respectively:

$$y = 0, \quad k_{se,1} \frac{dT_s}{dy} = h_c(T_s - T_c) \quad (8)$$

$$y = 0, \quad m c_f(T_f - T_c) = h_c(T_s - T_c) \quad (9)$$

$$y = H, \quad q = k_{se,2} \frac{dT_s}{dy} \quad (10)$$

At the interface of two layers,  $y = h$ , the solid temperature, fluid temperature and heat flux should be continuous, therefore the interface conditions are:

$$y = h, \quad T_s|_{y=h^-} = T_s|_{y=h^+} \quad (11)$$

$$y = h, \quad T_f|_{y=h^-} = T_f|_{y=h^+} \quad (12)$$

$$y = h, \quad k_{se,1} \frac{dT_s}{dy} \Big|_{y=h^-} = k_{se,2} \frac{dT_s}{dy} \Big|_{y=h^+} \quad (13)$$

Using the following dimensionless quantities:

$$Y = \frac{y}{H}, \quad \theta_s = \frac{T_s - T_c}{qH/k_f}, \quad \theta_f = \frac{T_f - T_c}{qH/k_f}$$

$$\kappa_i = \frac{k_{se,i}}{k_f}, \quad \lambda_{12} = \frac{k_{se,2}}{k_{se,1}}, \quad Bi_{v,i} = \frac{h_{v,i}H^2}{k_{se,i}}$$

$$Bi_c = \frac{h_cH}{k_{se,1}}, \quad \gamma = \frac{h}{H}$$

the governing equations and boundary conditions above can be rewritten as:

$$\frac{d^2 \theta_s}{dY^2} = Bi_{v,i}(\theta_s - \theta_f) \quad (14)$$

$$M \frac{d\theta_f}{dY} = Bi_{v,i}\kappa_i(\theta_s - \theta_f) \quad (15)$$

$$Y = 0, \quad \frac{d\theta_s}{dY} = Bi_c\theta_s \quad (16)$$

$$Y = 0, \quad M\theta_f = Bi_c\kappa_1\theta_s \quad (17)$$

$$Y = 1, \quad \kappa_2 \frac{d\theta_s}{dY} = 1 \quad (18)$$

$$Y = \gamma, \quad \theta_s|_{Y=\gamma^-} = \theta_s|_{Y=\gamma^+} \quad (19)$$

$$Y = \gamma, \quad \theta_f|_{Y=\gamma^-} = \theta_f|_{Y=\gamma^+} \quad (20)$$

$$Y = \gamma, \quad \frac{d\theta_s}{dY} \Big|_{Y=\gamma^-} = \lambda_{12} \frac{d\theta_s}{dY} \Big|_{Y=\gamma^+} \quad (21)$$

Wang and Wang [9] presented an analytical solution to solve the transpiration cooling problems of single layer. It is not difficult to extend the analytical solution into the problems with two porous layers, and the general solutions of governing equations (14)–(15) can be deduced as:

$$\begin{cases} \theta_{s,i} = K_{i,1}C_{i,1}e^{k_{i,1}Y} + K_{i,2}C_{i,2}e^{k_{i,2}Y} + C_{i,3} \\ \theta_{f,i} = C_{i,1}e^{k_{i,1}Y} + C_{i,2}e^{k_{i,2}Y} + C_{i,3} \end{cases} \quad (22)$$

Here,  $C_{i,j}$  ( $i = 1, 2, j = 1, 2, 3$ ) are integration constants, can be obtained by Eqs. (16)–(21),  $k_{i,(1,2)}$  and  $K_{i,(1,2)}$  can be calculated by:

$$k_{i,(1,2)} = \frac{-Bi_{v,i}\kappa_i/M \pm \sqrt{(Bi_{v,i}\kappa_i/M)^2 + 4Bi_{v,i}}}{2} \quad (23)$$

$$K_{i,(1,2)} = 1 + \frac{M}{Bi_{v,i}\kappa_i}k_{i,(1,2)} \quad (24)$$

Through the analytical solution, it can be validated that the solid temperature at the hot surface is dependent on the thermal conductivities, porosities of two layers and the thickness ratio of the first layer to the entire structure.

$$\theta_s(Y = 1) = f(\varepsilon_1, \varepsilon_2, k_{s,1}, k_{s,2}, \gamma) \quad (25)$$

If the target of structure optimization is to get the lowest temperature at the hot surface of the porous structure through a relationship similar to Eq. (25), obviously, it is difficult to optimize the structure by traditional methods.

### 3. Genetic algorithm (GA)

As an intelligent method to simulate the process of natural biotic inheritance and evolution, the GA has the characteristics of latent parallelism, non-differentiability of objective function, and its feasible solution is encoded into a chromosome which consists of several genes. Through the selection, crossover and mutation, the population of the chromosome is continuously evolved, finally converged to the chromosome which has high fitness, an optimal solution of the problem can be obtained thereby, this process is called an entire iteration process.

In this paper, five genes are used to characterize two porosities and two compositions of two layers, and one thickness ratio of the first layer to the entire structure respectively. The five genes are encoded as:

0011100|1100101|00|11|**0011110**

The first two sets of seven-bit code with underline are two genes which represent the porosities of each layer, and can be transformed into decimal value by the following formula:

$$\varepsilon = \frac{pr}{127}(\varepsilon_{\max} - \varepsilon_{\min}) + \varepsilon_{\min}$$

Table 1  
Thermal conductivity, relative density, cost and synthetic cost [5] of the three metals

Material	Thermal conductivity (W/mK)	Density/Al	Cost/Fe	Synthetic cost
Al	203.5	1	18.9	18.9
Cu	386.4	3.305	12.8	42.3
Fe	46.52	2.911	1	2.911

Here,  $pr$  is a decimal value between 0 and 127 which corresponds to a free porosity to vary between 0 and 1,  $\varepsilon_{\max}$  and  $\varepsilon_{\min}$  are the maximal and minimal value of the porosity, and given at 0.8 and 0.2, respectively, in this paper. A seven-bit code can approach to a precision of  $8 \times 10^{-3}$ , therefore the relative error of the porosity expression is less than 1% in this paper. The next two sets of italic code represent the compositions of each layer, 00 is aluminum Al, 01 is copper Cu, 10 is iron Fe, and 11 is a null number, respectively. The last boldface set of seven-bit binary code is the thickness ratio of the first layer to the entire structure. Although the porosity of packed beds only can vary between the limits 0.2595 and 0.4764, for analyzing the influence of the porosities of each porous layers, respectively, the maximum and minimum value of the ratio are limited to 0.8 and 0.2, respectively, in this work.

In order to get the optimal solution which satisfies the limitations of total weight and synthetic cost, two penalty coefficients,  $g_c$  and  $g_w$ , are introduced. The fitness of the chromosome can be calculated by:

$$f = a\theta + g_c[(C - C_0)/C_0] + g_w[(W - W_0)/W_0] \quad (26)$$

Here,  $C_0$  and  $W_0$  are the limitations of cost and weight respectively,  $a$  is an amplification coefficient. The values of those coefficients can be determined in the debug process of the GA program. The total weight of the porous structure is calculated by:

$$W(\text{Weight}) = (1 - \varepsilon_1) \times \rho_1 \times h + (1 - \varepsilon_2) \times \rho_2 \times (H - h) \quad (27)$$

The synthetic cost of the porous structure is computed by:

$$C(\text{Synthetic cost}) = (1 - \varepsilon_1) \times \rho_1 \times c_1 \times h + (1 - \varepsilon_2) \times \rho_2 \times c_2 \times (H - h) \quad (28)$$

In this paper, the chromosome number is a given value 500, the probabilities of the crossover and mutation are given at 0.8 and 0.1, respectively, the conception of steady reproduction without duplicate is employed as selection to avoid pre-maturity. Through a large numbers of numerical trials, it is found that once the iterative time is more than 500, the optimal results trends to a steady value, therefore each optimization result is created through 500 iterative times. Three metallic materials, aluminum, copper and iron, are selected as composites, and their thermal conductivity and relative density to aluminum at 300 K, relative cost and synthetic cost (cost  $\times$  density) to iron are listed in Table 1.

## 4. Results and discussions

### 4.1. Under constant pressure difference

Calculations are carried out with a total thickness of 0.1 m and a heat flux of  $1 \times 10^7$  W/m<sup>2</sup> s. At first, a pressure difference between the hot and cold sides of the porous structure is given with a certain Bejan number of  $Be = 10^{11}$ , here the momentum equation has to be solved. If the weight and cost of the structure are not restricted, the target of the optimization is to obtain the lowest temperature at the hot surface of the porous structure, and the five optimal results are shown in Table 2.

It is well known that a higher effective thermal conductivity allows the porous structure to conduct heat farther away from the hot side, and can enhance the heat exchange between the fluid coolant and solid structure within the pores. Therefore the results of Table 2 are not surprising, why the Cu with the highest thermal conductivity is the only choice of the porous media. Another influence factor on the optimal structure is the porosity of each layer. A higher porosity corresponds to a larger permeability and coolant mass flow rate, but a smaller effective thermal conductivity, and a variable volumetric heat transfer coefficient as shown in Fig. 2. Thus the porosity is a competing factor. From Table 2 it can be found that the porosities of two layers are distinct, and the porosity of the first layer reaches to the superior limit 0.8. This phenomenon can be explained by the fact that in the first layer, an increase in the coolant mass flow rate due to the larger porosity plays a more important role than the corresponding decrease in the effective thermal conductivity. The thickness of each layer is also a competing factor which influences the permeability and effective thermal conductivity synchronously, therefore it is reasonable that the each layer has different thickness.

Under the given pressure difference, the dimensionless coolant mass flow rates to get the corresponding the hot surface temperature are also listed in Table 2. It is noticeable that the mass flow rate of the 5th result is larger than that of the 3rd, but the temperature of the 5th is higher than that of the 3rd. The reason for this phenomenon is that in the computation with given pressure difference, the dimensionless coolant mass flow rate is not the optimized goal.

In the optimizations mentioned above, the entire weight and synthetic cost of the porous structure are not considered. In most situations, the cost of products is an important factor to be considered by fabricants. If the overall cost of the porous structure is restricted by 5.0, which is less than a half of the first result in Table 2, a new series of optimal results are listed in Table 3. It is clear that a combination of copper and iron makes the synthetic cost of the structure falls.

In the regions of spaceflight and aviation, the weight of the structure is usually an important factor to be considered. If the total weight of the porous structure is also restricted by 0.25 which is less than a half of the first result in Table 3, another series of optimal results are shown in Table 4. It is clear that the aluminum structure has less weight, and an acceptable cost.

Table 2  
Optimal solutions at  $Be = 10^{11}$ ,  $\varepsilon \in [0.2, 0.8]$ , without constraints of weight and cost

No.	First layer	Second layer	Temperature	Weight	Synthetic cost	Coolant ( $M$ )
1	Cu/0.8/0.7825	Cu/0.6819/0.2075	0.3479E-4	0.7428	9.51	41994
2	Cu/0.8/0.7975	Cu/0.6819/0.2025	0.3473E-4	0.7409	9.48	42246
3	Cu/0.8/0.8	Cu/0.6819/0.2	0.3466E-4	0.7391	9.46	42438
4	Cu/0.8/0.8	Cu/0.6488/0.2	0.3488E-4	0.7609	9.74	40232
5	Cu/0.8/0.795	Cu/0.6961/0.265	0.3487E-4	0.7524	9.63	43123

Note: Cu/0.6/0.7975 means the metal/porosity/thickness of the optimized structure.

Table 3  
Optimal solution with constraint cost < 5.0, at  $Be = 10^{11}$ ,  $\varepsilon \in [0.2, 0.8]$

No.	First layer	Second layer	Temperature	Weight	Synthetic cost	Coolant ( $M$ )
1	Fe/0.8/0.79	Cu/0.6819/0.21	0.3488E-4	0.7029	3.89	41088
2	Fe/0.8/0.8	Cu/0.6819/0.2	0.3480E-4	0.6760	3.16	42438
3	Fe/0.8/0.7925	Cu/0.6791/0.2075	0.3488E-4	0.6827	3.31	42062
4	Fe/0.8/0.8	Cu/0.6830/0.2	0.3480E-4	0.6752	3.15	42508
5	Fe/0.8/0.8	Cu/0.6819/0.2	0.3480E-4	0.6771	3.19	42438

Table 4  
Optimal solution with constraints weight < 0.25, cost < 5.0, at  $Be = 10^{11}$ ,  $\varepsilon \in [0.2, 0.8]$

No.	First layer	Second layer	Temperature	Weight	Synthetic cost	Coolant ( $M$ )
1	Al/0.8/0.795	Al/0.6813/0.2050	0.4132E-4	0.2246	4.25	42271
2	Al/0.8/0.8	Al/0.6801/0.2	0.4119E-4	0.2240	4.23	42323
3	Al/0.8/0.7925	Al/0.6789/0.2075	0.4132E-4	0.2254	4.26	42049
4	Al/0.8/0.7875	Al/0.6796/0.2125	0.4140E-4	0.2259	4.27	41976
5	Al/0.8/0.79	Al/0.6801/0.21	0.4137E-4	0.2255	4.26	42064

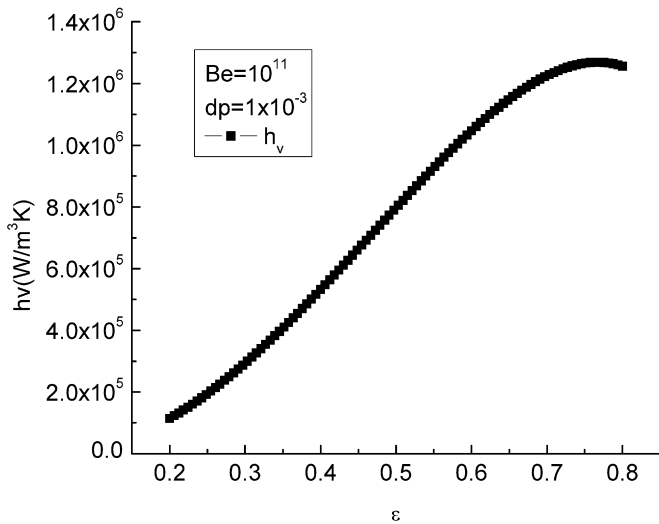


Fig. 2. Variation of volumetric heat transfer coefficient with porosity under given pressure difference.

It can be seen that the average cost of Table 3 and the average weight of Table 4 falls widely, but the average temperatures in Table 4 are much higher than those of Table 3. This phenomenon can be explained by Fig. 3, which shows the temperature distributions of the two-phase within the porous structures, and are obtained using the third data of Tables 3 and 4. The two series of data have the same thickness ratio and very close porosity, but different compositions. In Table 3, the composition of the first and second layer is Fe and Cu, respectively, and in

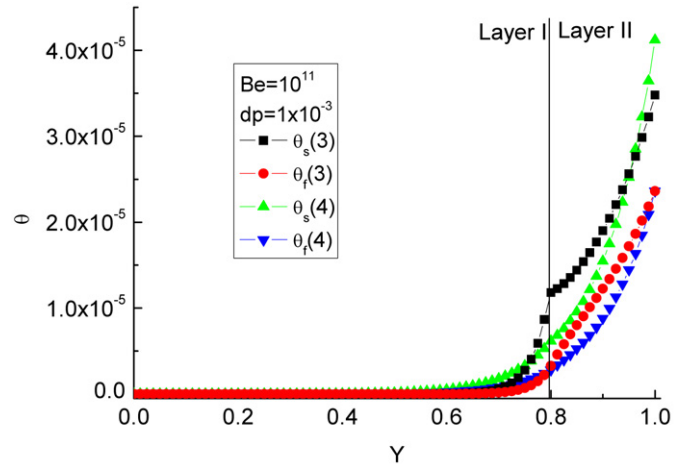


Fig. 3. Temperature distributions within Cu/Fe and Al/Al structures of the third data in Tables 3 and 4 under constant pressure difference.

Table 4 the structure is made of Al with two porosities. The thermal conductivity of Cu is larger than that of Al, and the thermal conductivity of Fe is less than that of Al. As shown in Fig. 3, in the second layer, the temperatures of the solid structure and fluid coolant decrease quickly in the opposite direction of  $Y$ , and the differences between solid and fluid temperatures in the first layer are much less than that in the second layer. Therefore the thermal diffusion of the porous structure and the heat exchange between the two-phase in the second layer is more important than that in the first layer. This is the reason, why Cu

Table 5  
Optimal solution without constraint, at  $M = 4E4$ ,  $\varepsilon \in [0.2, 0.8]$

No.	First layer	Second layer	Temperature	Weight	Synthetic cost	$Be$
1	Cu/0.3465/0.225	Cu/0.4078/0.775	0.3231E-4	2.0024	25.63	1.82E11
2	Cu/0.2898/0.245	Cu/0.4078/0.755	0.3231E-4	2.0516	26.26	2.44E11
3	Cu/0.3795/0.20	Cu/0.4078/0.80	0.3231E-4	1.9837	25.39	1.64E11
4	Cu/0.2047/0.305	Cu/0.4078/0.695	0.3231E-4	2.161	27.66	6.15E11
5	Cu/0.2898/0.24	Cu/0.4079/0.76	0.3231E-4	2.0498	26.24	2.42E11

Table 6  
Optimal solution with constraints weight < 2, cost < 15.  $M = 4E4$ ,  $\varepsilon \in [0.2, 0.8]$

No.	First layer	Second layer	Temperature	Weight	Synthetic cost	$Be$
1	Fe/0.4031/0.5025	Cu/0.4078/0.4975	0.3231E-4	1.85	13.40	1.58E11
2	Fe/0.4645/0.55	Cu/0.4031/0.4475	0.3232E-4	1.7451	12.22	1.25E11
3	Fe/0.5024/0.5025	Cu/0.4031/0.4975	0.3232E-4	1.6937	13.09	1.15E11
4	Fe/0.6157/0.5025	Cu/0.4031/0.4975	0.3232E-4	1.5436	13.12	9.51E10
5	Fe/0.7480/0.5125	Cu/0.4031/0.4875	0.3232E-4	1.3377	12.69	8.44E10

with a larger conductivity in the second layer can reach a better cooling effectiveness than Al.

#### 4.2. Constant coolant mass flow rate

When the coolant mass flow rate is given at  $M = 4E4$ , only the thermal governing equations have to be solved. At first, the weight and cost of the porous structure are not considered, only the composition, porosity and thickness are optimized, and five optimal results are shown in Table 5. It can be found that the optimal results are very different from that of Table 2. This is reasonable, because the porosity and thickness have not any effect on the coolant mass flow rate here, and only on the thermal conductivity and heat transfer coefficient. Although the temperatures of five results are very close, their pressure losses are very different, obviously the 3rd result is the best, because its Bejan number is the least, and this means that the pressure loss is the least.

Under the total weight and cost constraints of weight < 2 and cost < 15, a series of optimal results are arranged in Table 6. The fourth result with the lowest pressure loss is the best. It is noticeable that the composition of the second layer in Tables 5 and 6 is the same Cu, the porosities of the second layer in Tables 5 and 6 are also very close, although the porosity of the first layer is widely different from each other, and the hot surface temperatures of Tables 5 and 6 are very close. From this fact, a conclusion can be drawn: under the condition of constant coolant mass flow rate, the influence of the porosity of the first layer on the hot surface temperature is relatively small. It is interesting to notice that the temperatures in Tables 6 and 5 are almost the same, but the combinations of Fe–Cu are not detected when the weight and cost constraints are not considered. This phenomenon can be explained by that if the optimization process is free, the composition of the porous matrix will be firstly locked at the material with the highest thermal conductivity to accelerate the convergence process. In actual fact, the actual temperatures in Table 6 are slightly higher than those in Table 5.

## 5. Conclusions

In this paper, the temperature at the hot surface of the porous structure is obtained with an analytical solution of simplified local thermal non-equilibrium model, the lowest temperature at the hot surface is the target of transpiration cooling structure, and the composition, porosity and thickness of each layer are optimized by the GA under the conditions of total weight and cost constraint. Through the discussions of the results, the following conclusions can be drawn:

- (1) The coupled method of the genetic algorithm with the analytic solution is usable to design the multi-layered porous structures of transpiration cooling. The target of the optimization can be the lowest temperature at the hot surface, of cause also the minimal pressure drop, or the least coolant mass flow rate.
- (2) Under different constrains, the optimal porosities, materials and thicknesses of each layer are different. When the pressure difference is constant, the porosity of the first layer has notable effect on the coolant mass flow rate and should be large possible, whereas when the coolant mass flow rate is constant, the porosity of the first layer has minor effect on the hot surface temperature.
- (3) Under the conditions of constant pressure or constant mass flow rate, the composition and porosity of the second layer have more important effects on the hot surface temperature than those of the first layer.

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